MATH 121A Prep: Bases

Facts to Know:

$$A = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_m \end{bmatrix}$$

Let $\vec{v_1}, \dots, \vec{v_m}$ be vectors in a subspace $V \subset \mathbb{R}^n$, and A the $n \times m$ matrix with columns $\vec{v_1}, \dots, \vec{v_m}$ $\vec{v_1}, \dots, \vec{v_m}$ are Linearly Independent if: $\vec{v_1} + \vec{v_2} \vec{v_1} + \dots + \vec{v_m} = \vec{o}$ thus $\vec{c_1} = \vec{c_2} = \dots = \vec{o}$.

 \leftarrow A $\dot{c} = \dot{o}$ having only the trivoch solution ($\ddot{c} = \ddot{o}$) $\vec{v_1}, ..., \vec{v_m} \operatorname{Span} V \text{ if: } \forall \vec{w} \in V$ therease $C_1, ..., C_n \in S_n$. $\vec{w} = C_1 \vec{v_1} + C_2 \vec{v_2} + ... + C_m \vec{v_m}$

← Ac = is has a solution

 $\vec{v_1}, \dots, \vec{v_m}$ is a Basis for V if: Hey are linearly independent and Hey span V.

Dimension of a Subspace V: (1) Frad basis \overline{V}_{i}

(2) Dinenson of V = m = # of rectors in

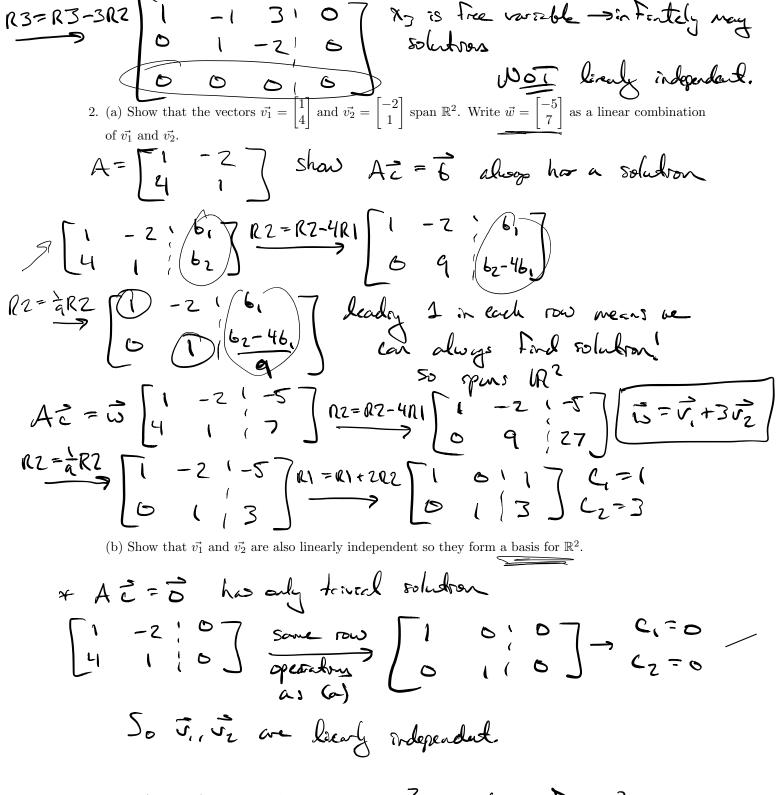
Examples:

1. Determine whether the vectors $\vec{v_1} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$, $\vec{v_3} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ are linearly independent.

A= [2 6 27 wis A = = = = = has shy trivial 2 1 0] Solution.

$$\begin{bmatrix} 2 & 0 & 2 & 0 \\ 1 & -1 & 3 & 0 \\ 2 & 1 & 0 & 0 \end{bmatrix} \xrightarrow{\text{NIZ-RZ}} \begin{bmatrix} 1 & -1 & 3 & 0 \\ 2 & 0 & 2 & 0 \\ 2 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{R2 = R7 - 7R1}{R3 = R3 - 2R1} \begin{bmatrix} 1 & -1 & 3 & 6 \\ 6 & 2 & -4 & 6 \\ 0 & 3 & -6 & 6 \end{bmatrix} \xrightarrow{R2 = \frac{1}{2}R7} \begin{bmatrix} 1 & -1 & 3 & 6 \\ 6 & 1 & -2 & 6 \\ 0 & 3 & -6 & 6 \end{bmatrix}$$



Linearly independent + pour IR² -> basis for IR²

(IR² has dinension 2)

